NONDESTRUCTIVE MEASUREMENT OF MAGNETIC PERMEABILITY

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INTRODUCTION

Conventional techniques for measuring magnetic permeability using permeameters require either a thin strip or a ring to be cut from a sample of the material. Obviously, this type of measurement is destructive in nature and cannot be used for *in-situ* permeability measurements. In this paper, we describe a technique that can be used to measure the permeability of flat plates nondestructively. The method uses a closed-form solution for the on-axis field that is transmitted through the ferromagnetic plate by an axisymmetric coil with a rectangular cross-section, energized by a DC (or very low frequency) current.

THEORY

In Ref. 1 Dodd and Deeds presented an exact solution for the case of a delta coil (single loop) above a two-layer conducting medium, see Figure 1. Solving for the vector

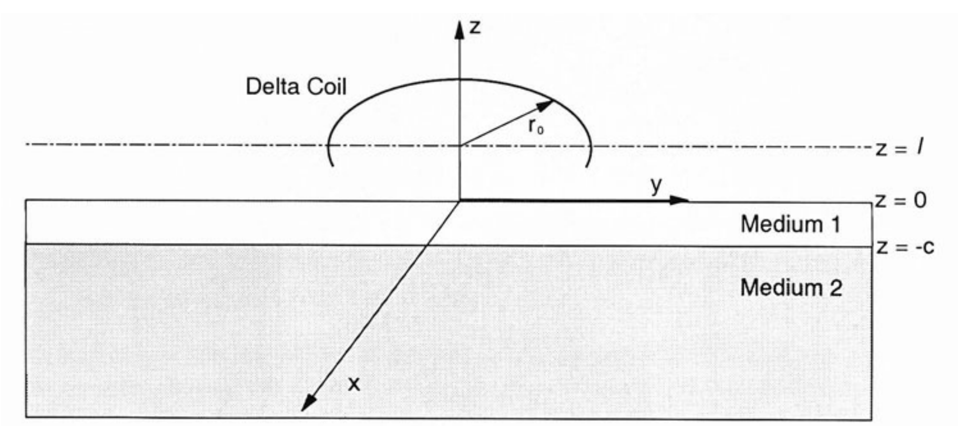

Figure 1. Sketch of a delta coil above two conductors.

potential, a^i (r,z), and assuming axial symmetry and time harmonic current, they used separation of variables to obtain the solution which is now well known

$$a^1(r,z) = \int_0^\infty c_2^1(\alpha)e^{-\alpha_0 z}J_1(\alpha r)d\alpha \quad , \tag{1}$$

and

$$a^2(r,z) = \int_0^\infty \left[c_3^2(\alpha)e^{+\alpha_0 z} + c_2^2(\alpha)e^{-\alpha_0 z}\right]J_1(\alpha r)d\alpha \quad , \tag{2}$$

$$a^3(r,z) = \int_0^\infty \left[c_3^3(\alpha)e^{+\alpha_1 z} + c_2^3(\alpha)e^{-\alpha_1 z}\right]J_1(\alpha r)d\alpha \quad , \tag{3}$$

$$a^4(r,z) = \int_0^\infty c_3^4(\alpha)e^{+\alpha_2 z}J_1(\alpha r)d\alpha \quad . \tag{4}$$

$$c_2^1(\alpha) = \mu_0 I(\alpha)\frac{\alpha}{2\alpha_0}\left\{e^{\alpha_0 \ell} + \left[\frac{(\alpha_0+\beta_1)(\beta_1-\beta_2) + (\alpha_0-\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}{(\alpha_0-\beta_1)(\beta_1-\beta_2) + (\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}\right]e^{-\alpha_0 \ell}\right\} \quad , \tag{5}$$

$$c_3^2(\alpha) = \mu_0 I(\alpha)\frac{\alpha}{2\alpha_0}e^{-\alpha_0 \ell} \quad , \tag{6}$$

$$c_2^2(\alpha) = \mu_0 I(\alpha)\frac{\alpha}{2\alpha_0}\left\{\frac{(\alpha_0+\beta_1)(\beta_1-\beta_2) + (\alpha_0-\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}{(\alpha_0-\beta_1)(\beta_1-\beta_2) + (\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}\right\}e^{-\alpha_0 \ell} \quad , \tag{7}$$

$$c_3^3(\alpha) = \mu_0 I(\alpha)\left\{\frac{\alpha(\beta_1+\beta_2)e^{-\alpha_0 \ell + 2\alpha_1 c}}{(\alpha_0-\beta_1)(\beta_1-\beta_2) + (\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}\right\} \quad , \tag{8}$$

$$c_2^3(\alpha) = \mu_0 I(\alpha)\left\{\frac{\alpha(\beta_1-\beta_2)e^{-\alpha_0 \ell}}{(\alpha_0-\beta_1)(\beta_1-\beta_2) + (\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}\right\} \quad , \tag{9}$$

$$c_3^4(\alpha) = \mu_0 I(\alpha)\left\{\frac{2\alpha\beta_1 e^{(\alpha_2+\alpha_1)c}e^{-\alpha_0 \ell}}{(\alpha_0-\beta_1)(\beta_1-\beta_2) + (\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}}\right\} \quad . \tag{10}$$

Here, as usual, ω represents the harmonic frequency of the current, σ_i and μ_i are the conductivity and permeability of the i^{th} medium, and

$$\alpha_i = \sqrt{\alpha^2 - \omega^2\mu_i\epsilon_i + j\omega\mu_i\sigma_i} \quad , \tag{11}$$

$$\beta_i = \frac{\mu_o}{\mu_i}\alpha_i \quad . \tag{12}$$

$I(\alpha)$ in the above expressions represents the Hankel transform of the current distribution.

In the following, for simplicity, we replace the c_j^i's with P_i's which are related to the c_j^i's by

$$P_1 = c_2^1 \frac{e^{-\alpha_0 z}}{\alpha} \tag{13}$$

$$P_2 = \left(c_3^2 + c_2^2\right)\frac{e^{-\alpha_0 z}}{\alpha} \tag{14}$$

$$P_3 = \left(c_3^3 + c_2^3\right)\frac{e^{-\alpha_0 z}}{\alpha} \tag{15}$$

$$P_4 = c_3^4 \frac{e^{-\alpha_0 z}}{\alpha} \quad . \tag{16}$$

Closed Form Solutions

If we assume medium 2 in Figure 1 is air, and we restrict ourselves to $\omega = 0$ (DC current), then

$$P_4(\alpha, z) = \frac{2\mu}{\alpha(\mu + 1)^2} \frac{e^{\alpha(2c + z - \ell)}}{e^{2\alpha c} - g} \quad , \tag{17}$$

where we have introduced $\mu = \frac{\mu_1}{\mu_0}$ and $g = \left(\frac{\mu - 1}{\mu + 1}\right)^2$. The fundamental solution ("delta coil") for the vector potential transmitted thought the plate is obtained by assuming the current is concentrated in a single ring above the plate and applying Eq. 4

$$a^4(r,z) = \mu_o Ir_o \frac{2\mu}{(\mu+1)^2} \int_o^\infty \frac{e^{\alpha(2c+z-\ell)}}{e^{2\alpha c}-g} J_1(\alpha r_o)J_1(\alpha r)\, d\alpha \quad . \tag{18}$$

Converting this expression to one for the z-component of B requires applying $\underset{\sim}{\nabla}\Lambda\underset{\sim}{a} = \underset{\sim}{B}$ and produces

$$B_z^4(r,z;r_o,l) = 2\mu_o Ir_o \frac{\mu}{(\mu+1)^2} \int_o^\infty \frac{e^{\alpha(2c+z-\ell)}}{e^{2\alpha c}-g} \alpha J_1(\alpha r_o)J_o(\alpha r)d\alpha \tag{19}$$

for the magnetic field on the side of the plate opposite the coil (transmitted field). Taking $r = 0$ (center of coil) in this expression then gives

$$B_z^4(0,z;r_o,l) = 2\mu_o Ir_o \frac{\mu}{(\mu+1)^2} \int_o^\infty \frac{\alpha e^{\alpha(2c+z-\ell)} J_1(\alpha r_o)}{e^{2\alpha c}-g} d\alpha \quad . \tag{20}$$

To evaluate this inverse Hankel transform we extended a result due to Kapteyn (2)

$$B_z^4(0,z;r_o,l) = 2\mu_o Ir_o^2 \frac{\mu}{(\mu-1)^2} \sum_{n=1}^\infty \frac{g^n}{\left\{r_o^2 + [2c(n-1)+\ell-z]^2\right\}^{3/2}} , \tag{21}$$

$$(\ell \geq c, \; z \leq -c) \quad .$$

This is the required closed-form expression. We can now use superposition to determine the transmitted field for any DC, axisymmetric current distribution. For applications it is helpful to normalize this expression by taking the ratio of this field and that obtained in air. The field in air can be derived from Eq. 21, or as is well known

$$B_z^{air} = \frac{\mu_o Ir_o^2}{2\left[r_o^2 + (\ell-z)^2\right]^{3/2}} \quad . \tag{22}$$

So the normalized field for the delta coil becomes

$$B_z^N(0,z;r_o,\ell) = 4\left[r_o^2 + (\ell-z)^2\right]^{3/2} \frac{\mu}{(\mu-1)^2} \sum_{n=1}^\infty \frac{g^n}{\left\{r_o^2 + [2c(n-1)+\ell-z]^2\right\}^{3/2}} \quad . \tag{23}$$

One practical extension of the above result can be made for coils with a rectangular cross section. Figure 2 defines the geometry. The current distribution generated by a rectangular coil can be written as

$$I(r,z) = \left[H(r-r_i) - H(r-r_o)\right]\left[H(z-s_\ell) - H(z-s_u)\right] \quad . \tag{24}$$

Using superposition Eq. (21) gives

$$B_z^4(0,z) = 2\mu_o I \frac{\mu}{(\mu - 1)^2} \int_{r_i}^{r_o} \int_{s_\ell}^{s_u} \sum_{n=1}^{\infty} \frac{g^n r^2}{\{r^2 + [2c(n-1) + (\ell - z)]^2\}^{3/2}} \, dr \, d\ell \quad , \tag{25}$$

for the field transmitted by the rectangular coil. To evaluate this expression we first interchange the orders of integration and summation

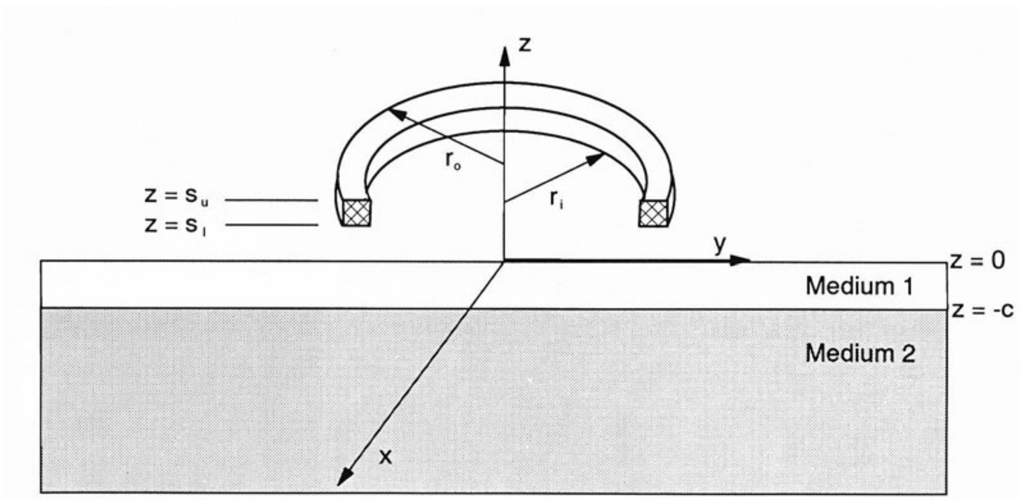

Figure 2. Sketch of rectangular coil above two conductors.

$$B_z^4(0,z) = 2\mu_o I \frac{\mu}{(\mu - 1)^2} \sum_{n=1}^{\infty} g^n \int_{s_\ell}^{s_u} \int_{r_i}^{r_o} \frac{r^2}{\{r^2 + [2c(n-1) + (\ell - z)]^2\}^{3/2}} dr \, d\ell \quad . \tag{26}$$

The inner integral can be done easily, leaving

$$B_z^4(0,z) = \frac{2\mu_o I \mu}{(\mu - 1)^2} \sum_{n=1}^{\infty} g^n \int_{s_\ell}^{s_u} \frac{r_i}{\sqrt{r_i^2 + [2c(n-1) + (\ell - z)]^2}}$$

$$- \frac{r_o}{\sqrt{r_o^2 + [2c(n-1) + (\ell - z)]^2}} + \ln \left\{ \frac{r_o + \sqrt{r_o^2 + [2c(n-1) + (\ell - z)]^2}}{r_i + \sqrt{r_i^2 + [2c(n-1) + (\ell - z)]^2}} \right\} d\ell \quad . \tag{27}$$

This integral can also be easily done and leads to

$$B_z^4(0,z) = 2\mu_o I \frac{\mu}{(\mu-1)^2} \sum_{n=1}^{\infty} g^n *$$

$$\left[[(s_u - z) + 2c(n-1)] \ln \left\{ \frac{r_o \sqrt{r_o^2 + (s_u - z)^2 + 4c(n-1)[c(n-1) + s_u - z]}}{r_i + \sqrt{r_i^2 + (s_u - z)^2 + 4c(n-1)[c(n-1) + (s_u - z)]}} \right\} \right. \tag{28}$$

$$\left. -[(s_\ell - z) + 2c(n-1)] \ln \left\{ \frac{r_o + \sqrt{r_o^2 + (s_\ell - z)^2 + 4c(n-1)[c(n-1) + (s_\ell - z)]}}{r_i + \sqrt{r_i^2 + (s_\ell - z)^2 + 4c(n-1)[c(n-1) + (s_\ell - z)]}} \right\} \right] ,$$

as the field transmitted by the rectangular coil. Again it is helpful to normalize by the field "in air" (take $\mu = 1$ in Eq. 28)

$$B_z^{air}(0,z) = \frac{\mu_o I}{2} \left\{ (z - s_\ell) \ln \left[\frac{r_o + \sqrt{r_o^2 + (s_\ell - z)^2}}{r_i + \sqrt{r_i^2 + (s_\ell - z)^2}} \right] - (z - s_u) \ln \left[\frac{r_o + \sqrt{r_o^2 + (s_u - z)^2}}{r_i + \sqrt{r_i^2 + (s_u - z)^2}} \right] \right\} . \tag{29}$$

Verification of the Solution Using Finite Elements

To verify that Eqs. 21 and 28 describe the relationship between the field transmitted through a ferromagnetic plate and the permeability of the plate, finite element solutions were made for several cases. Figure 3 shows the calculated transmittance for a delta-coil for permeabilities ranging from 50 to 300. Here, the plate thickness was 0.26 inches, while the coil radius was 2.00 inches. The exact solution and the finite element solution agree closely with the maximum difference being about 5 percent. Figure 4 shows the same information for a coil with a rectangular cross-section. Here the plate thickness is

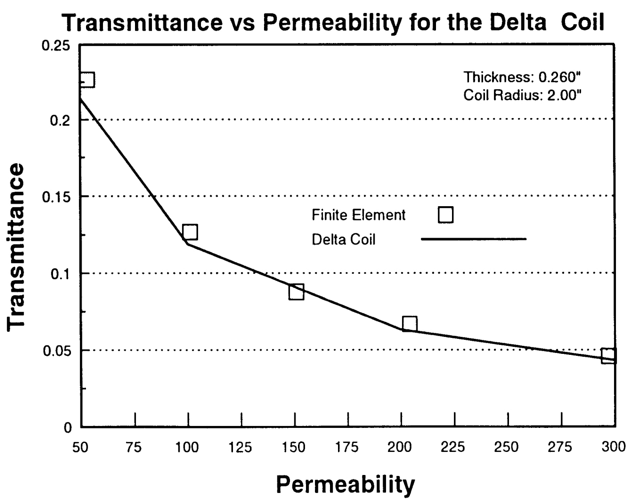

Figure 3. Comparison of the exact solution for transmittance for a delta coil with a finite element solution, as a function of permeability.

0.03 inches. The coil inner radius is 2.0 inches, with an outer radius of 2.2 inches and thickness of 0.2 inches. Again the agreement between the exact solution and the finite element solution is quite good, with a maximum difference less than 1 percent. Note also that the transmittance approaches one as the material becomes nonmagnetic.

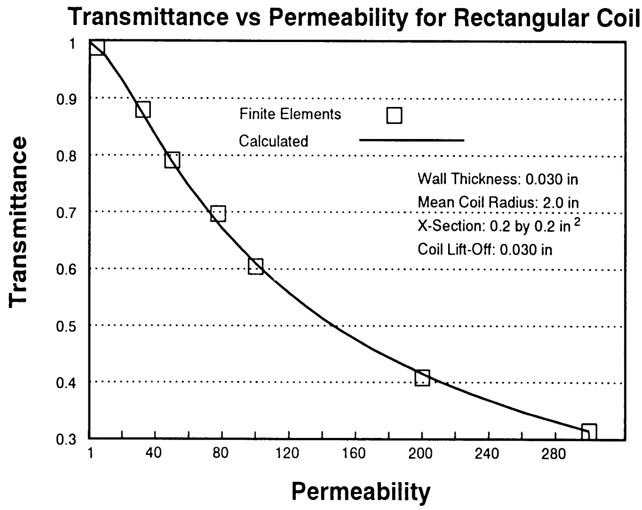

Figure 4. Comparison of the exact solution for transmittance for a rectangular coil with a finite element solution, as a function of permeability.

APPLICATION

Magnetic saturation is often interpreted as the point where the incremental permeability, defined as the ratio of the change in the magnetic flux density, B, for some small change in the magnetizing field, H, becomes unity (in cgs units). Using the method described above, the permeability of gas transmission pipeline steel (grade X52) was measured as the material was magnetized. Later, a magnetization curve was measured for the same material using conventional (destructive) techniques and the incremental permeability was computed from the magnetization curve. Note that the two methods measure the permeability in two different directions, since the magnetization curve is measured in the plane of the material while the method described here measures primarily through the thickness. The results for the two different methods of measuring permeability are shown in Figure 5. The longitudinal (or in-plane) incremental permeability approaches 1 at a magnetizing field of about 900 Oe. The transverse permeability, on the other hand, is still about 70 for this value of H, and it dropped only to a value of 33 for a magnetizing field as large as 5000 Oe.

The eddy current skin depth was measured in the same material as a function of the magnetization by comparing the amplitude of a defect signal to the defect depth. These measurements indicated that the transverse permeability, as measured by the method presented here, was a better indicator of the skin depth than the incremental permeability.

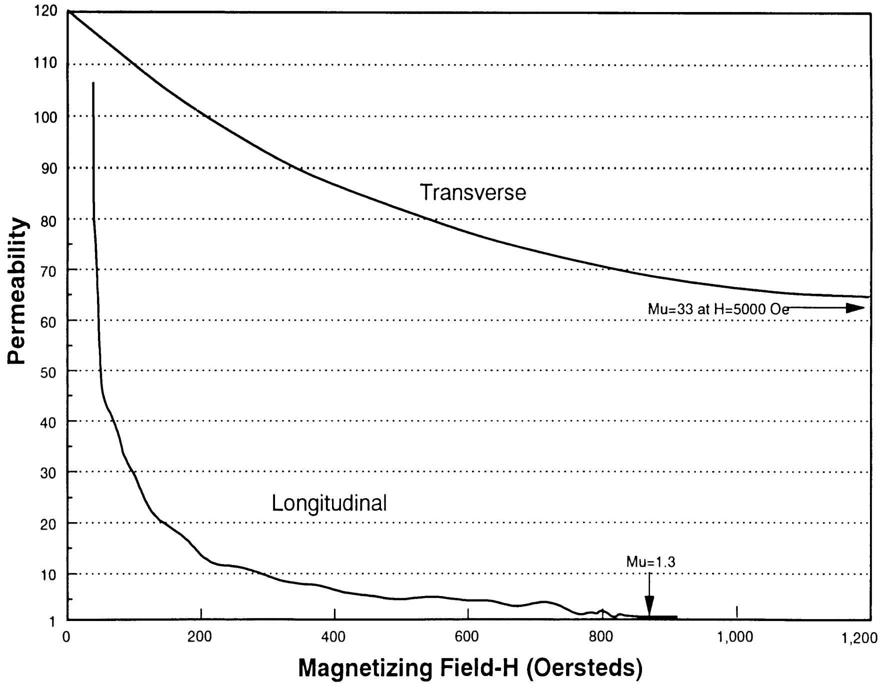

Figure 5. Comparison of the transverse and longitudinal permeability for X52 pipeline steel.

CONCLUSIONS

A nondestructive method of measuring the permeability of ferromagnetic plates has been developed using an exact solution for the magnetic field transmitted through the plate at low frequencies. Using this method, the permeability of X52 pipeline steel was measured as a function of the magnetization level in the steel. Both these measurements and eddy current measurements of skin depth showed that the permeability was still quite high even though the incremental permeability was essentially one.

REFERENCES

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